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Some problems of parallelization of process of the decision of algebraic systems
ABSTRACT
The abstract of dissertation is presented for the scientific degree of the PhD at the speciality of 6D060100 - Mathematics

Importance of research. The beginnings of 50 years of the XX century numerical methods quickly developed. Especially it is noticeable in the development of applied packages for the decision of linear algebraic systems such as EISPACK and LINPACK. In the given applied packages engineers use ready modules for drawing up difficult structures at the solution of necessary tasks in the area.

The problem of solving large-dimensional linear algebraic systems is reduced to increasing computing power while optimizing the calculation process. In some cases, the number of parameters is so large that a single-processor system lacks the necessary memory or requires an impractically long computation time, making the solution process more complex. Therefore, the use of parallel algorithms becomes essential.

Such problems arise in many scientific fields, including quantum physics (small particle physics, nuclear physics, quantum field theory), molecular physics, quantum chemistry, biology, ecology, Earth sciences (atmospheric physics, meteorology, climatology, ocean physics), economics and econometrics (computational economics, macroeconomics, optimal control theory), mathematical linguistics (speech recognition, text analysis, and machine translation), computer science (database processing, image recognition), social sciences, hydrodynamics, gas dynamics, medicine, pharmaceuticals, and more. Today, parallel algorithms in computations significantly reduce the time required for computational tasks.

It should be noted that a sequential algorithm does not always yield the desired result on a multiprocessor system. This is due to the system's architecture and necessitates the development of specialized parallel algorithms. Parallel algorithms are particularly common in numerical series computations in discrete mathematics, combinatorics, linear algebra, grid-based calculations, and graph-related problems.

Currently, the most powerful supercomputer is located at the Lawrence Livermore National Laboratory (LLNL) in California, USA. Named El Capitan, this multiprocessor system has a capacity of 1.742 EFlop/s, or 1.74 quintillion floating-point operations per second. The system consists of over 11 million cores. El Capitan is used for nuclear arsenal simulation and security, as well as for research in high-energy density physics and the development of new materials.

In the early 2000s, GPUs began to be used for parallel computing. Graphics processing units (GPUs) significantly accelerate the solution of large linear equation

systems due to their thousands of parallel cores. Unlike central processing units (CPUs), they efficiently handle large matrices and vectors using high-speed memory (HBM, GDDR6). GPUs are widely employed in linear algebra methods (LU decomposition, QR decomposition, conjugate gradient method) and are implemented through CUDA (NVIDIA) and ROCm (AMD) frameworks.

The aim of the PhD thesis. Construction methods of parallelization of the process of the decision of linear and nonlinear algebraic systems.

The object of investigation. Linear algebraic systems of the equations, nonlinear algebraic systems of the equations.

Methods of the investigation. At the decision of the system of the linear algebraic equations such as

$$Ax = f$$

transition to the approximate solution of an equivalent task on minimization of values of functionality of

$$J(x) = |Ax - f|^2.$$

In many cases instead of finding of the exact solution of the equation, it is used

$$Ax = f.$$

It is expedient to minimize a difference

$$|Ax - f|.$$

According to this principle, cases when the matrix of A is well conditioned and reversible are considered. The approximate method of the decision and the corresponding parallel algorithm is given in the thesis. If the matrix of A is badly caused (i.e. it is singular or its singular values are very small) the minimized values of the functionality of Tikhonov for $\varepsilon \geq 0$

$$J_\varepsilon(x) = |Ax - f|^2 + \varepsilon|x|^2.$$

In a case when the matrix of A is singular, there are some decisions of which the decision with the smallest norm gets out.

Scientific novelty of the investigation. The thesis contains new results in the field of parallelization of the process of the decision of algebraic systems.

The theoretical and practical importance of the investigation. The thesis contains theoretical and practical values. The received results allow to solve approximately linear systems with a reversible or irreversible matrix. The case of a nonlinear system is also investigated.

Publications.

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The structure and volume of the dissertation. The dissertation consists of 5 chapters (each chapter consists of several parts), introduction, conclusion, list of references and appendices. 81 — total of pages.